Author: Dennis Linn Situation 1: Squaring a Binomial

Prompt

In an Integrated Math 3 course, students were solving radical equations. The teacher wrote $\sqrt{x} = x - 2$ on the board. A suggestion came from the students to square both sides to eliminate the radical sign from the left side of the equality. A portion of the students wrote on their paper $x = x^2 - 4$. Several students wrote $x = x^2 + 4$.

Commentary

Students seem to recognize that the distributive property should be used when two distinct binomials such as (x + a)(x + b) show up in assorted problems. And possibly will think of the distributive property when (x + a)(x + a) shows up as well. But when $(x + a)^2$ happens to appear in a problem, there remains a tendency is to square the binomial as if it were the sum of the squares of the two terms and write $x^2 + a^2$, or as the case may be, $x^2 - a^2$. Students have been squaring binomials since a very early age when they use the multiplication algorithm to square a two-digit number as developed in Foci 1. They can also use geometry to calculate the area of a square with sides of (x + a) as in Foci 2.

Mathematical Foci

Mathematical Focus 1

The process of squaring a binomial is equivalent to using the multiplication algorithm that is used for squaring a two-digit number.

When 32^2 is to be evaluated, the process many are taught a method that looks similar to the following.

```
  32
* 32
 ===
  64
+960
 ===
1024
```

The algorithm above starts by first multiplying the two in the second line by the 32 in the top line. This 64 is written in the third line. Then the 3 in the second line, which represents 30, is multiplied by the 32 in the top line; this product of 96 is sometimes written as 960 instead of 96. If is written as 96, there is a “blank” space used in place of that zero. The addition of 64 and 960 yields a sum of 1024.

The process could be written out horizontally as the sum of all four of the respective products.

(2 * 2) + (2 * 30) + (30 * 2) + (30 * 30) =
4 + 60 + 60 + 900 =
64 + 960 =
1024.

This is equivalent to a distributive process that the student may have already used to multiply (x + a)(x + a).

(30 + 2)(30 + 2) =
(30 * 30) + (30 * 2) + (30 * 2) + (2 * 2) =
900 + 60 + 60 + 4 =
960 + 64 =
1024.

As the two digit numbers get larger, and the carrying process gets more involved, but the process remains the same.

```
   86
 * 86
  ===
  516
+6880
  ===
 7396
```

The above is equivalent to:

(6 * 6) + (6 * 80) + (80 * 6) + (80 * 80) =
36 + 480 + 480 + 6400 =
516 + 6880 =
7396.

And it is also equivalent to:

(80 + 6) (80 + 6) =
(80 * 80) +(80 * 6) + (80 * 6) + (6 * 6) =
6400 + 480 + 480 + 36 =

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$6880 + 516 =$
$7396.$

One could think of the two digit number above as TU, where T is in the tens place of a decimal number and U is in the units place; thus making the value of TU equal to (T * 10 + U) or (10T + U).

$(10T + U)(10T + U) =$
$(100T^2 + 10T * U + 10U * T + U^2) =$
$100T^2 + 2*10T*U + U^2.$

Using this last expression for T = 3 and U = 2 (as in the first example above) yields

$(100 * 3^2) + (2*10*3*2) + (2^2) =$
$900 + 2*60 + 4 =$
$900 + 120 + 4 =$
$1024.$

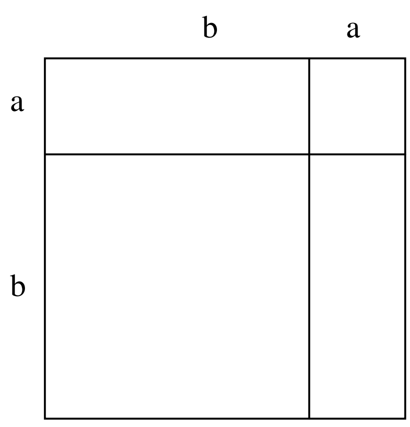
Thus squaring (a + b) involves the same process that is used to square a two digit number.

$(a +b)(a + b) =$
$a^2 + ab + ba + b^2 =$
$a^2 + 2ab + b^2$

Mathematical Focus 2

The process of squaring a binomial is equivalent to finding the area of a square of length (a+ b).

Let each side of the square be x units in length such that x = a + b.



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So the area of the square is $x^2 = (a + b)^2$.

The area of the square is also equal to the sum of the four subdivisions, that is the sum of the two rectangles plus the sum of the two squares. The two rectangles both have identical areas of ab, and the two squares have areas of a^2 and b^2. Thus their sum is $a^2 + 2ab + b^2$.

If a = 30 and b = 2, then the area of the two rectangles plus the area of the two squares would yield the following

$30^2 + (30 * 2) + (30 * 2) + 2^2 =$
900 + 60 + 60 + 4 =
960 + 64 =
964.

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